MGSC 1205

Quantitative Methods I

Slides Five – Multi-period application

& Sensitivity Analysis

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Multi-period Applications: Production Scheduling

- Most challenging application of LP is modeling multi-period scenarios.
  - Situations where decision maker has to determine optimal decisions for several periods (weeks, months, etc.).
  - These problems especially difficult because decision choices in later periods are directly *dependent* on decisions made in earlier periods.
- Production planning must consider four factors:
  - Satisfying demands [simplifies planning & the scheduling of workers & machines]

$$\begin{bmatrix}\text{inventory}\\ \text{at the}\\ \text{end of}\\ \text{this month}\end{bmatrix} = \begin{bmatrix}\text{inventory}\\ \text{at the}\\ \text{end of}\\ \text{last month}\end{bmatrix} + \begin{bmatrix}\text{current}\\ \text{month's}\\ \text{production}\end{bmatrix} - \begin{bmatrix}\text{sales(demand)}\\ \text{this month}\end{bmatrix}$$

  - Necessity to reduce inventory carrying, or holding, costs. [suggests producing in each month only what is needed in that month]
  - Warehouse limitations cannot be exceeded. [without great additional storage cost]
  - Minimum production capacity should be used each month

Example: Greenberg Motors, Inc. (p. 104)

- **<u>Greenberg Motors, Inc.</u>** manufactures two different electrical motors for sale under contract to Drexel Corp: Model GM3A & GM3B
- **Demand** (Four-month order schedule for motors)

Model	January	February	March	April
GM3A	800	700	1,000	1,100
GM3B	1,000	1,200	1,400	1,400

- **Production cost**
  - Now GM3A $10, GM3B $6; Will increase in March to $11 and $6.60, respectively
- **Holding cost** (cost of carrying inventory)
  - GM3A is $0.18 & GM3B is $0.13 per month.
- **Storage capacity:** GMI can hold a maximum of 3300 motors of either type (they are similar in size) at any time
- **Each GM3A motor produced requires 1.3 hours of labour, and each GM3B takes a worker 0.9 hours to assemble.**
- **Minimum labour hours:** GMI has a base employment level of 2240 hours per month.
- **Maximum labor hours:** In a busy period, the company can bring two retired skilled employees on board to increase capacity to 2560 hours per month.
- **The company looks for** the production schedule that minimizes total cost of per unit production & monthly holding

Decision variables & Objective function

Decision Variables.

- P_{Ai} = number of model GM3A motors produced in month *i*
- P_{Bi} = number of model GM3B motors produced in month *i*
- I_{Ai} = level of on-hand inventory for GM3A at the end of month *i*
- I_{Bi} = level of on-hand inventory for GM3B at the end of month *i*

(*i*=1,2,3,4 for January–April).

- **Minimize total cost**

- **Production cost**
  - Now GM3A \$10, GM3B \$6; Will increase in March to \$11 and \$6.60, respectively

$PC = 10\,(P_{A1}+P_{A2})+11(P_{A3}+P_{A4}) + 6\,(P_{B1}+P_{B2})+6.60(P_{B3}+P_{B4})$

- **Holding cost** (cost of carrying inventory)
  - GM3A is \$0.18 & GM3B is \$0.13 per month.

$HC = 0.18(I_{A1}+I_{A2}+I_{A1}+I_{A2}) + 0.13(I_{B1}+I_{B2}+I_{B1}+I_{B2})$

- **Total cost = PC + HC**

Decision variables & Objective function

- **The objective function**

- **Minimize total costs**

$$C = 10\,(P_{A1}+P_{A2})+11(P_{A3}+P_{A4}) + 6\,(P_{B1}+P_{B2})+6.60(P_{B3}+P_{B4})$$
$$+\ 0.18(I_{A1}+I_{A2}+I_{A1}+I_{A2}) + 0.13(I_{B1}+I_{B2}+I_{B1}+I_{B2})$$

Constraints on Demand

Model	January	February	March	April
GM3A	800	700	1,000	1,100
GM3B	1,000	1,200	1,400	1,400

- Demand / Inventory constrains set the relationship between closing inventory this month, closing inventory last month, this month's production and sales.
- Inventory at end of month is:

$$\begin{pmatrix}\text{inventory}\\ \text{at the}\\ \text{end of}\\ \text{this month}\end{pmatrix} = \begin{pmatrix}\text{inventory}\\ \text{at the}\\ \text{end of}\\ \text{last month}\end{pmatrix} + \begin{pmatrix}\text{current}\\ \text{month's}\\ \text{production}\end{pmatrix} - \begin{pmatrix}\text{sales}\\ \text{to Drexel}\\ \text{this month}\end{pmatrix}$$

- $I_{A1} = 0 + P_{A1} - 800$ or $P_{A1} - I_{A1} = 800$ January GM3A demand
- $I_{B1} = 0 + P_{B1} - 1000$ or $P_{B1} - I_{B1} = 1000$ January GM3B demand

Constraints on Demand

Model	January	February	March	April
GM3A	800	700	1,000	1,100
GM3B	1,000	1,200	1,400	1,400

$$P_{A2} + I_{A1} - I_{A2} = 700$$ February GM3A demand

$$P_{B2} + I_{B1} - I_{B2} = 1,200$$ February GM3B demand

$$P_{A3} + I_{A2} - I_{A3} = 1,000$$ March GM3A demand

$$P_{B3} + I_{B2} - I_{B3} = 1,400$$ March GM3B demand

$$P_{A4} + I_{A3} - I_{A4} = 1,100$$ April GM3A demand

$$P_{B4} + I_{B3} - I_{B4} = 1,400$$ April GM3B demand

- If Greenberg wants to have on hand additional 450 GM3As and 300 GM3Bs at end of April, add constraints:

$I_{A4} = 450$ and $I_{B4} = 300$

Warehouse Space Constraints

- The storage area for GM can hold a maximum of 3,300 motors of either type at any one time.

$$I_{A1} + I_{B1} \leq 3,300$$

$$I_{A2} + I_{B2} \leq 3,300$$

$$I_{A3} + I_{B3} \leq 3,300$$

$$I_{A4} + I_{B4} \leq 3,300$$

Labour Constraints

- Minimum labour hours is 2,240 each month.
- The maximum labour capacity is 2,560 hours each month.
- GM3A requires 1.3 hours, GM3B requires 0.9 hours.

$$1.3P_{A1} + 0.9P_{B1} \geq 2,240$$ (January min. hours)

$$1.3P_{A1} + 0.9P_{B1} \leq 2,560$$ (January max. hours)

$$1.3P_{A2} + 0.9P_{B2} \geq 2,240$$ (February min. hours)

$$1.3P_{A2} + 0.9P_{B2} \leq 2,560$$ (February max. hours)

$$1.3P_{A3} + 0.9P_{B3} \geq 2,240$$ (March min. hours)

$$1.3P_{A3} + 0.9P_{B3} \leq 2,560$$ (March max. hours)

$$1.3P_{A4} + 0.9P_{B4} \geq 2,240$$ (April min. hours)

$$1.3P_{A4} + 0.9P_{B4} \leq 2,560$$ (April max. hours)

Solution to Multiple period applications

- Decision choices in later periods are directly dependent on decisions made in earlier periods.
- Decision variables: number of units produced/inventoried
- Objective: minimize cost
  - Total cost = production cost + holding cost (carrying inventory)
- **Constrains**
  - Inventory / Demand :

$$\begin{bmatrix}\text{inventory}\\ \text{at the}\\ \text{end of}\\ \text{this month}\end{bmatrix} = \begin{bmatrix}\text{inventory}\\ \text{at the}\\ \text{end of}\\ \text{last month}\end{bmatrix} + \begin{bmatrix}\text{current}\\ \text{month's}\\ \text{production}\end{bmatrix} - \begin{bmatrix}\text{sales(demand)}\\ \text{this month}\end{bmatrix}$$

  - Warehouse space: maximum storage area
  - Labor: minimum & maximum capacity

Marketing Application: Media Selection

- Win Big Gambling Club promotes gambling junkets from a large Midwestern city to casinos in the Bahamas.
- Club has budgeted up to $8,000 per week for local advertising.
- Money is to be allocated among four promotional media:

Medium	Audience Reached Per Ad	Cost Per Ads	Maximum Ads Per Week
TV spots	5000	$800	12
Newspaper ads	8500	$925	5
Prime-time radiospots	2400	$290	25
Afternoon radio spots	2800	$380	20

✓Contract arrangements require at least 5 radio spots per week.

✓Management insists no more than $1,800 be spent on radio ad per week.

Win Big's goal – reach largest possible high-potential audience through various media.

How can we handle changes?

- We have solved *LP* problems under **deterministic** assumptions.
  - find an optimum solution given certain constant parameters (costs, price, time, etc)
- How well do we know these parameters?
  - Usually not very accurately – rough estimates
  - Conditions in most world situations are dynamic & changing
    - ✓ prices of raw materials change
    - ✓ product supply changes
    - ✓ new machinery is bought to replace old
    - ✓ employee turnover occurs …

Sensitivity Analysis

- Post-optimality analysis: examining changes after the optimal solution has been reached.
  - input data are varied to assess optimal solution sensitivity.
- **Basic Question:** How does our solution change as the input parameters change?
  - How much does the objective function change?
  - How much do the optimal values of the decision variables change?
  - **Do our results remain valid** (If the parameters change...)?

Example: High Note Sound Company

- The company Manufactures quality CD players and stereo receivers.
- Each CD player sold results in $50 profit, while each receiver yields $120 profit.
- Each product requires skilled craftsmanship.
  - **Each CD player requires:** 2 hours electrician's time and 3 hours technician's time
  - **Each receiver requires:** 4 hours electrician's time and 1 hour technician's time
- **Hours available:** 80 for electrician's time, 60 for technician's time
- **Objective:** maximize profit

D6 =SUMPRODUCT(B5:C5,B6:C6)

High Note Sound Company

	C	R			
	CD players	Stereo receivers			
Solution value	0.00	20.00			
Profit	$50	$120	$2,400.00	<--	
Constraints					
Electricians' Time	2	4	80.00	<=	80
Audio Technicians' Time	3	1	20.00	<=	60
			LHS	**Sign**	**RHS**

B5:C5

D6

D8:D9

4.1 / Answer Report / Sensitivity Report

Ready NUM

Answer Report

Optimal Objective function value

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- ❖ This table gives us information obtained from the objective function.
  - ✓ which cell the objective function is located in
  - ✓ its initial value before solver was initiated
  - ✓ the value obtained by plugging in the values of the decision variables from the optimal corner point.

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.0[illegible]

Optimal solution values

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- This table gives us information on the decision variables.
  - ✓ which cells the decision variables are located in
  - ✓ their initial values before solver was initiated
  - ✓ their values corresponding to the optimal solution

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- This table gives us information the constrains.
  - ✓ Cell value: how much of the given resource is used up in obtaining the optimal solution
  - ✓ Formula: the constraint equation in cell notation

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

This column indicates whether a constraint is exactly satisfied (*LHS*=*RHS*)

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- **Binding** means the constrain is exactly satisfied, and *LHS* = *RHS*.
  - All the available resource is fully used in the solution
  - Nonbinding means that some of the resource has not been fully used up in the final solution

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final V
$B$5	Solution value CD players	0.00	
$C$5	Solution value Stereo receivers	0.00	

This column indicates the amount of unused resource

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- **Slack** is the difference between the *RHS* and the *LHS* of a ≤ constrain
  - Binding constrain: slack =0.
  - A nonbinding constrain is when the *slack* > 0.

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Electricians' Time	80.00	30.00	80.00	160.00	80.00
$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

- We are analyzing only one change at a time
- This table presents information regarding the impact of changes to the *OFC*s (i.e., the unit profits of $50 & $120)
  - ✓ **Allowable Increases & Allowable Decreases**: they are the range of values for which we can change the *OFC*s, and still have current **Corner Point** remain as **Optimal Solution**
  - **This is the whole point of doing the analysis!**

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Electricians' Time	80.00	30.00	80.00	160.00	80.00
$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

- ✓ It is allowed to increase the *OFC* value by up to $10 and still have no change in optimal solution.
- ✓ Excel's notation for ∞: the price of CD players can be dropped by ∞ and still has no change in optimal solution (we still won't make any).

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Electricians' Time	80.00	30.00	80.00	160.00	80.00
$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

At what point would we want to start making CD player?

- Back to **Reduced Cost**.
  - Losing $10 for each CD player that we choose to make with its current *OFC*.
  - If we can somehow raise the OFC by $10 per unit, making CD players stops losing money; If we raise it more than $10, then producing CD players becomes profitable
  - **This is exactly the Allowable Increase for CD player!**

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Electricians' Time	80.00	30.00	80.00	160.00	80.00
$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

- ✓ **Allowable Increases** & **Allowable Decreases**: they are the range of values for which we can change the *OFC*s, and still have current **Corner Point** remain as **Optimal Solution**

Sensitivity Analysis

- We have solved LP problems under deterministic assumptions. i.e., finding an optimum solution given certain constant parameters (costs, price, time, etc.)
- Conditions in most world situations are dynamic & changing
  - prices of raw materials change, product supply changes, new machinery is bought to replace old, employee turnover occurs …
- Post-optimality analysis: examining changes after the optimal solution has been reached.
- **Basic Question: How does our solution change as the input parameters change?**
  - **Do our results remain valid (If the parameters change...)?**